Lecture 8:

Wavefront Sensing

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Outline of lecture

- General discussion: Types of wavefront sensors

- Three types in more detail:
  - Shack-Hartmann wavefront sensors
  - Curvature sensing
  - Pyramid sensing

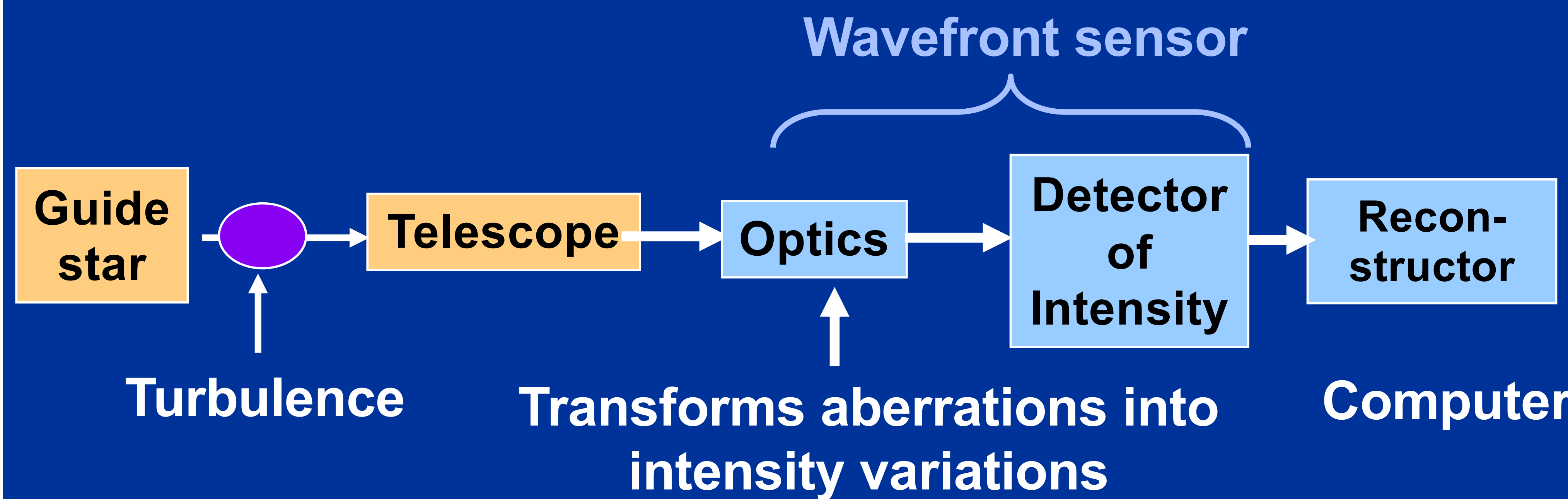
At longer wavelengths, one can measure phase directly

- FM radios, radar, radio interferometers like the VLA, ALMA
- All work on a narrow-band signal that gets mixed with a very precise "intermediate frequency" from a local oscillator. "Heterodyne" measurement.
- Very hard to do this at visible and near-infrared wavelengths
  - Could use a laser as the intermediate frequency, but would need tiny bandwidth of visible or IR light

Thanks to Laird Close's lectures for making this point

At visible and near-IR wavelengths, measure phase via intensity variations

- Difference between various wavefront sensor schemes is the way in which phase differences are turned into intensity differences
- General box diagram:



How to use intensity to measure phase?

- Irradiance transport equation: *A* is complex field amplitude, z is propagation direction. (Teague, 1982, JOSA 72, 1199)

$$\text{Let } A(x, y, z) = [I(x, y, z)]^{1/2} \exp[ik\phi(x, y, z)]$$

- Follow *I (x,y,z)* as it propagates along the z axis (paraxial ray approximation: small angle w.r.t. z)

$$\frac{\partial I}{\partial z} = -\nabla I \bullet \nabla \phi - I \nabla^2 \phi$$

Wavefront curvature: Curvature Sensors

Wavefront tilt: Hartmann and Pyramid sensors

Types of wavefront sensors

- **"Direct" in pupil plane:** split pupil up into subapertures in some way, then use intensity in each subaperture to deduce phase of wavefront.
  - Slope sensing: Shack-Hartmann, Pyramid sensing
  - Curvature sensing
- **"Indirect" in focal plane:** wavefront properties are deduced from whole-aperture intensity measurements made at or near the focal plane. Iterative methods – calculations take longer to do.
  - Image sharpening
  - Phase diversity, phase retrieval, Gerchberg-Saxton (these are used, for example, in JWST)
  - Usually used to measure nearly static aberrations

How to reconstruct wavefront from measurements of local "tilt"

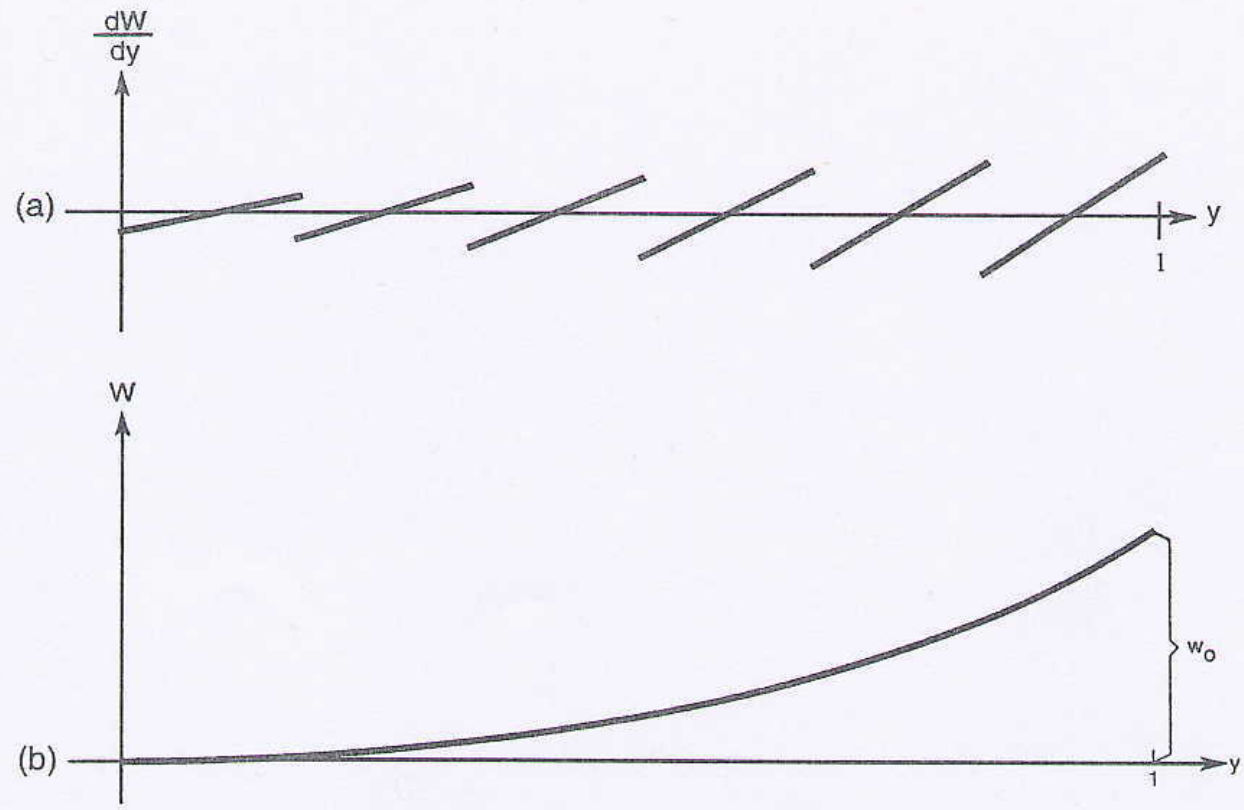
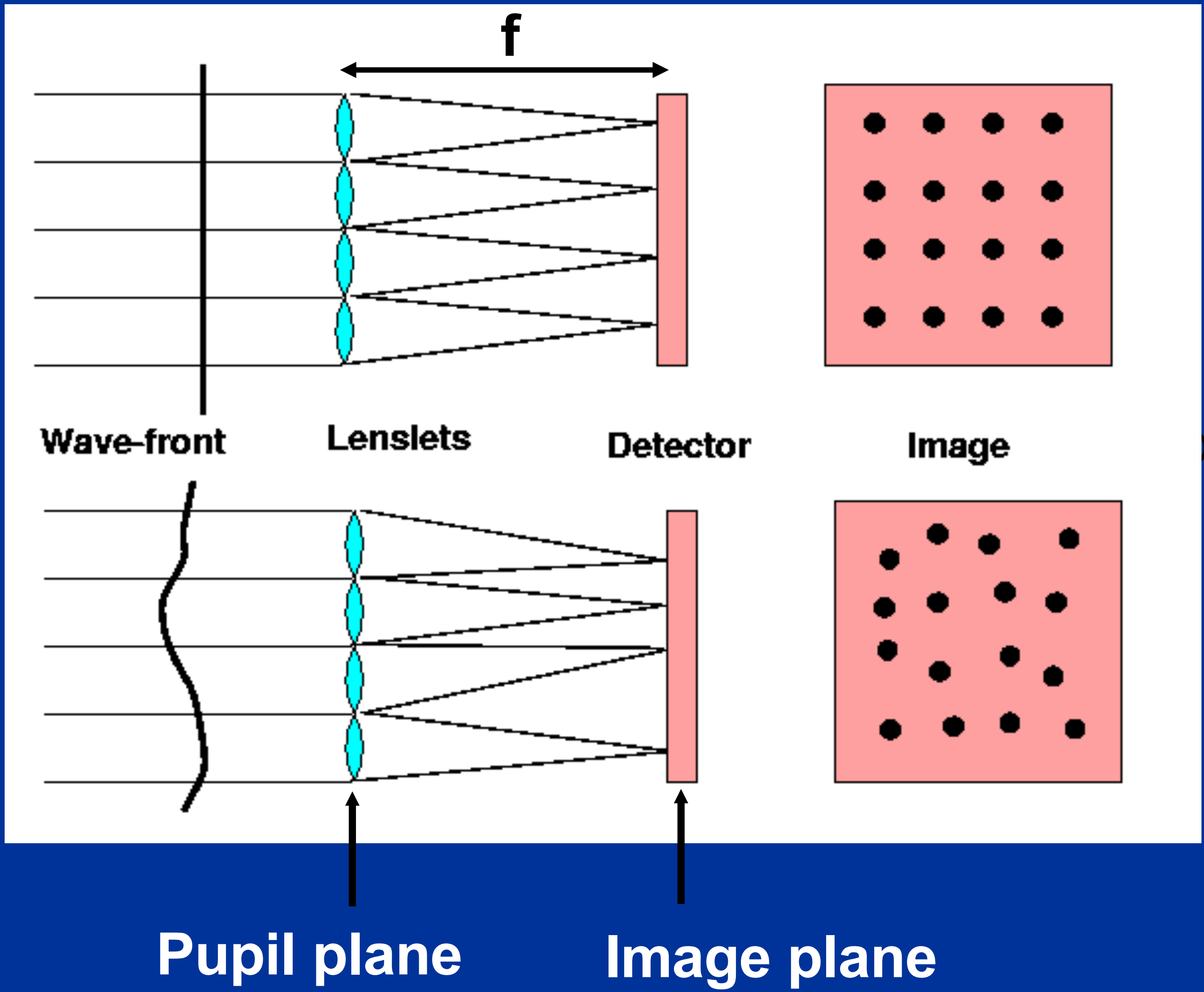


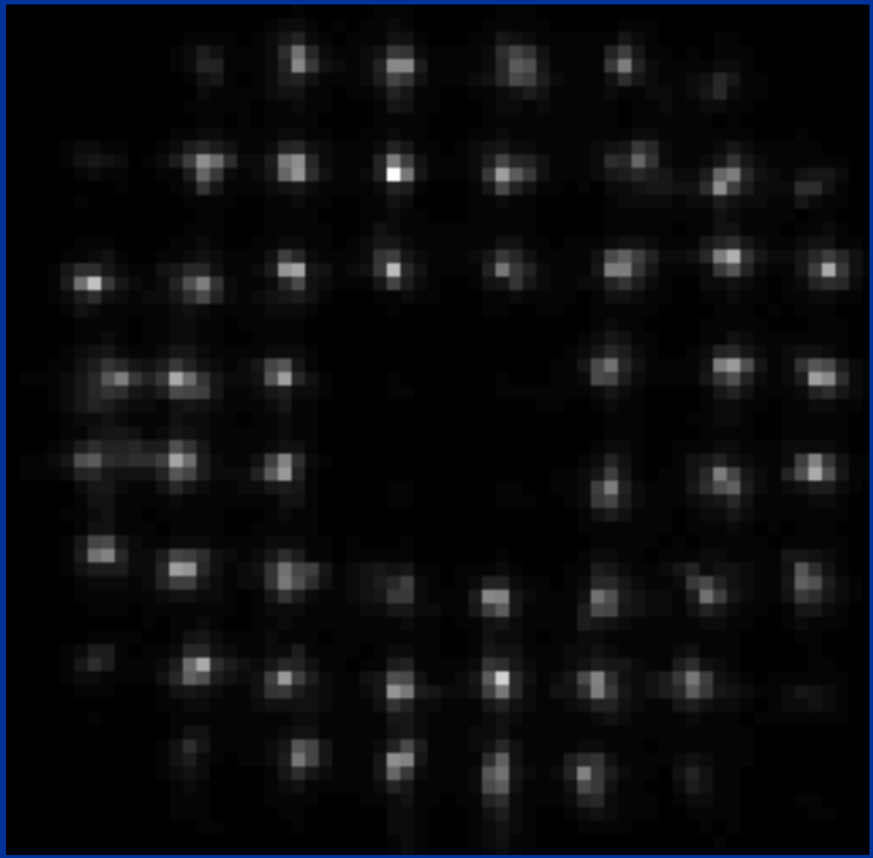
Figure 7 (a) Local tilt as a function of sampling location in pupil; (b) reconstructed wavefront estimate.

Shack-Hartmann wavefront sensor concept - measure subaperture tilts



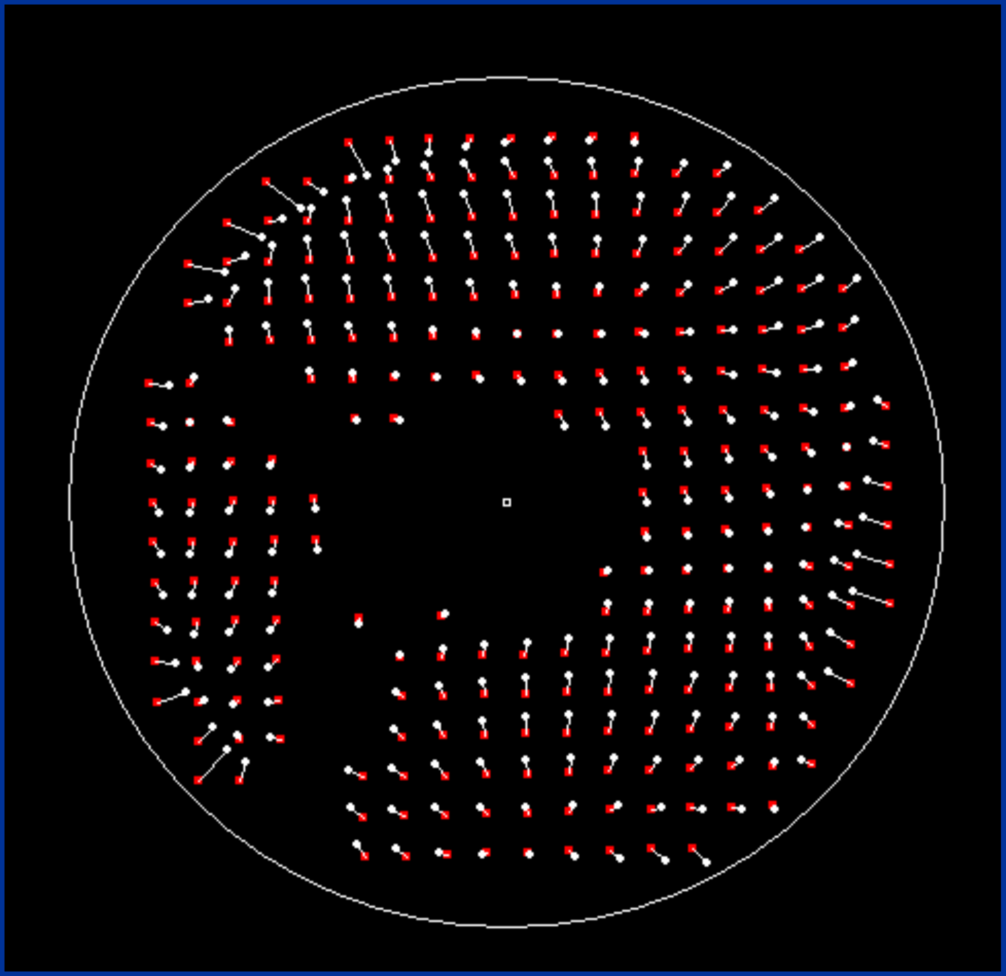
Credit:
A. Tokovinin

Example: Shack-Hartmann Wavefront Signals



Credit: Cyril Cavadore

Displacement of centroids



- Definition of centroid

$$\bar{x} \equiv \frac{\iint I(x,y)\, x\, dxdy}{\iint I(x,y) dxdy}$$

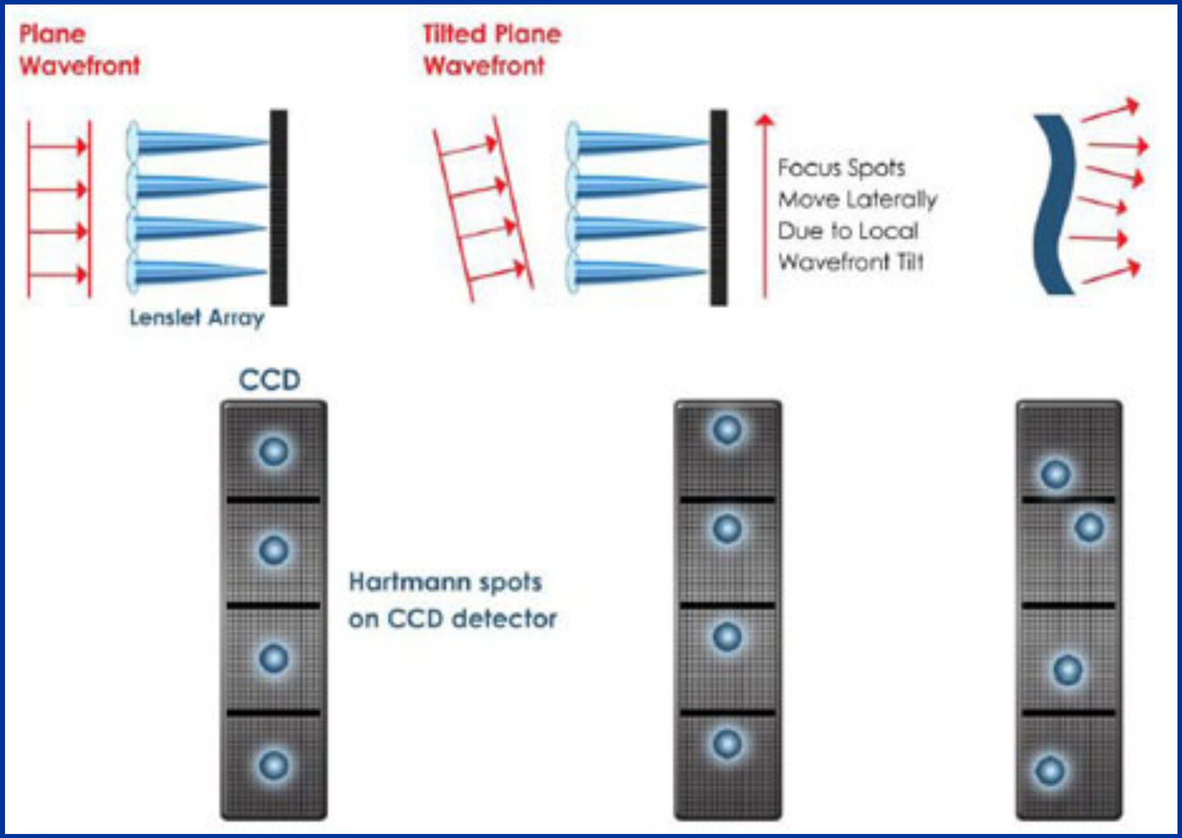
$$\bar{y} \equiv \frac{\iint I(x,y)\, y\, dxdy}{\iint I(x,y) dxdy}$$

- Centroid is intensity weighted

← Each arrow represents an offset proportional to its length

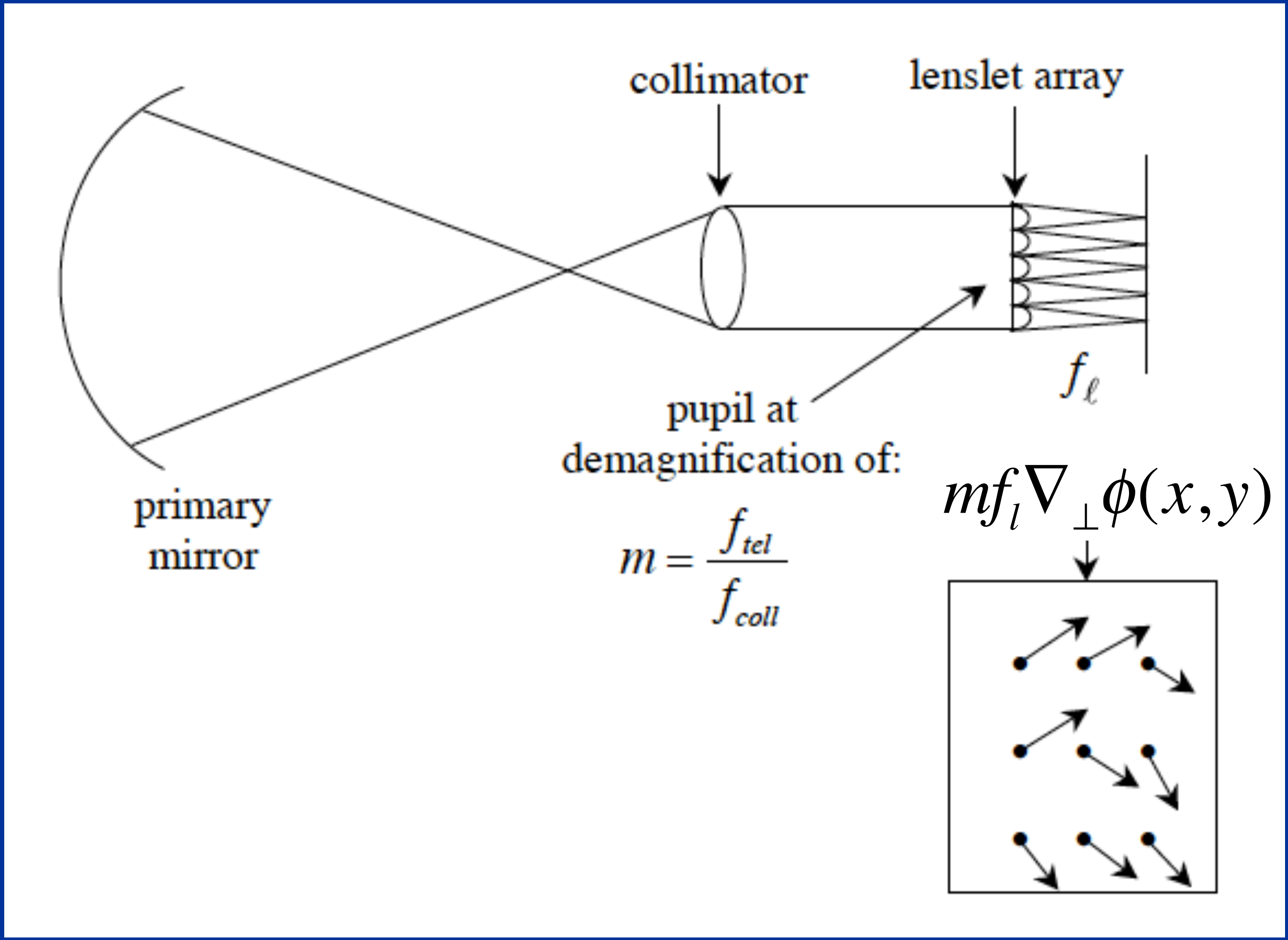
Credit: Cyril Cavador

Notional Shack-Hartmann Sensor spots



Credit: Boston Micromachines

Displacement of Hartmann Spots



Quantitative description of Shack-Hartmann operation

- Relation between displacement of Hartmann spots and slope of wavefront:

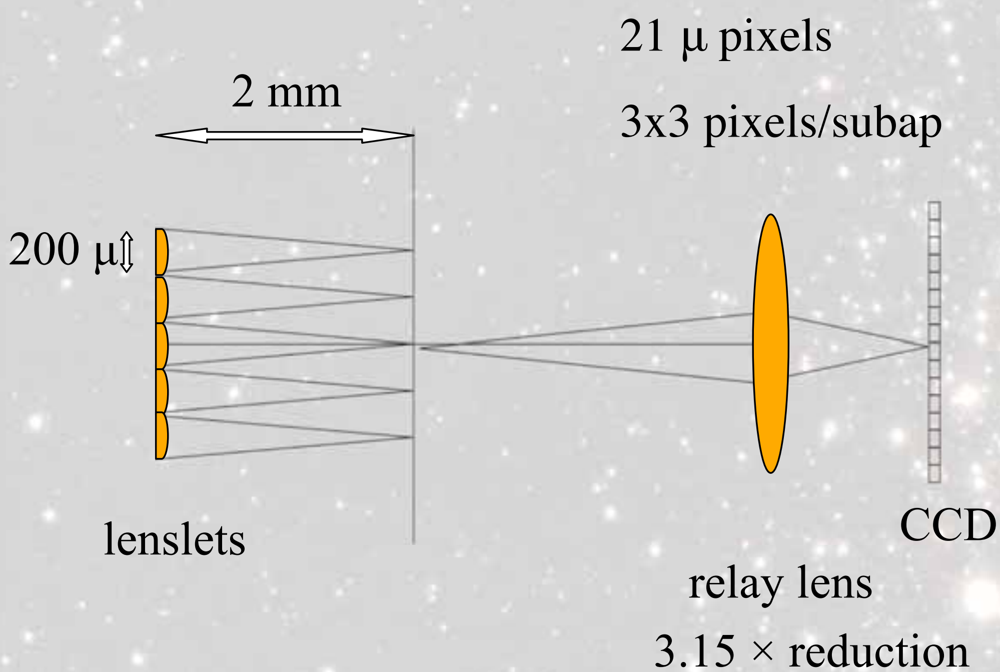
$$\Delta\vec{x} \propto \nabla_{\perp}\phi(x,y)$$

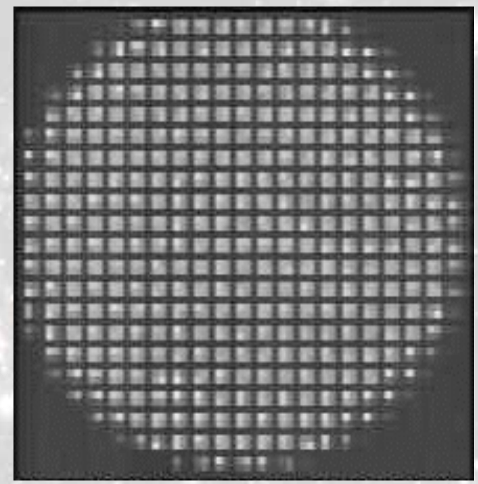
$$k\Delta\vec{x} = M f \nabla_{\perp}\phi(x,y)$$

where $k = 2\pi / \lambda$, Δx is the lateral displacement of a subaperture image, M is the (de)magnification of the system, f is the focal length of the lenslets in front of the Shack-Hartmann sensor

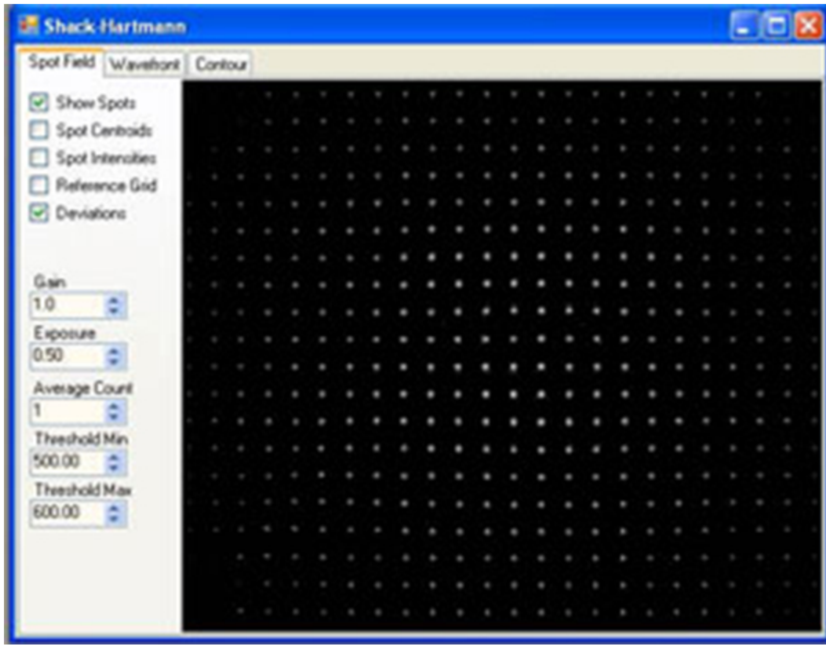
Typical Astronomy WFS

Former Keck AO WFS sensor

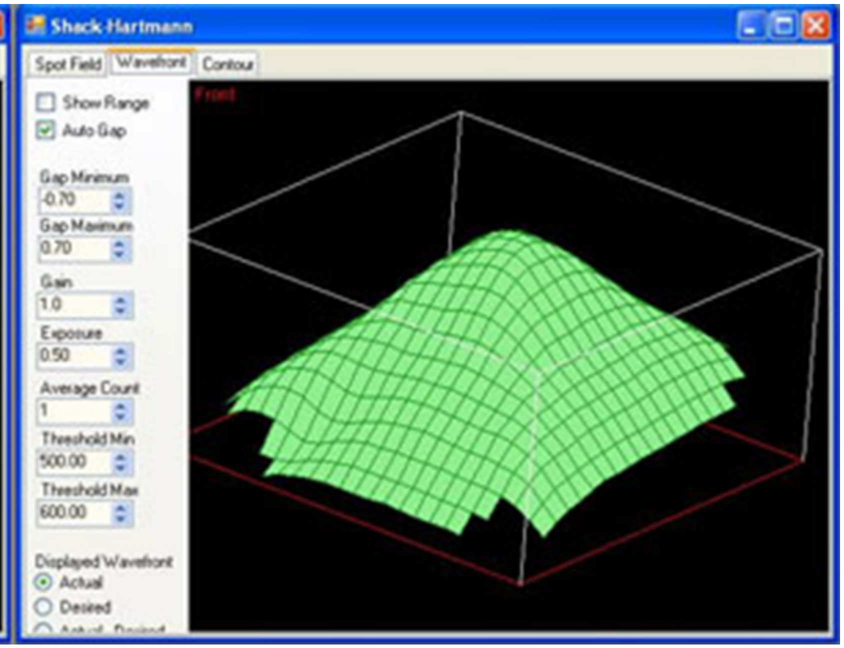




Credit: Marcos van Dam



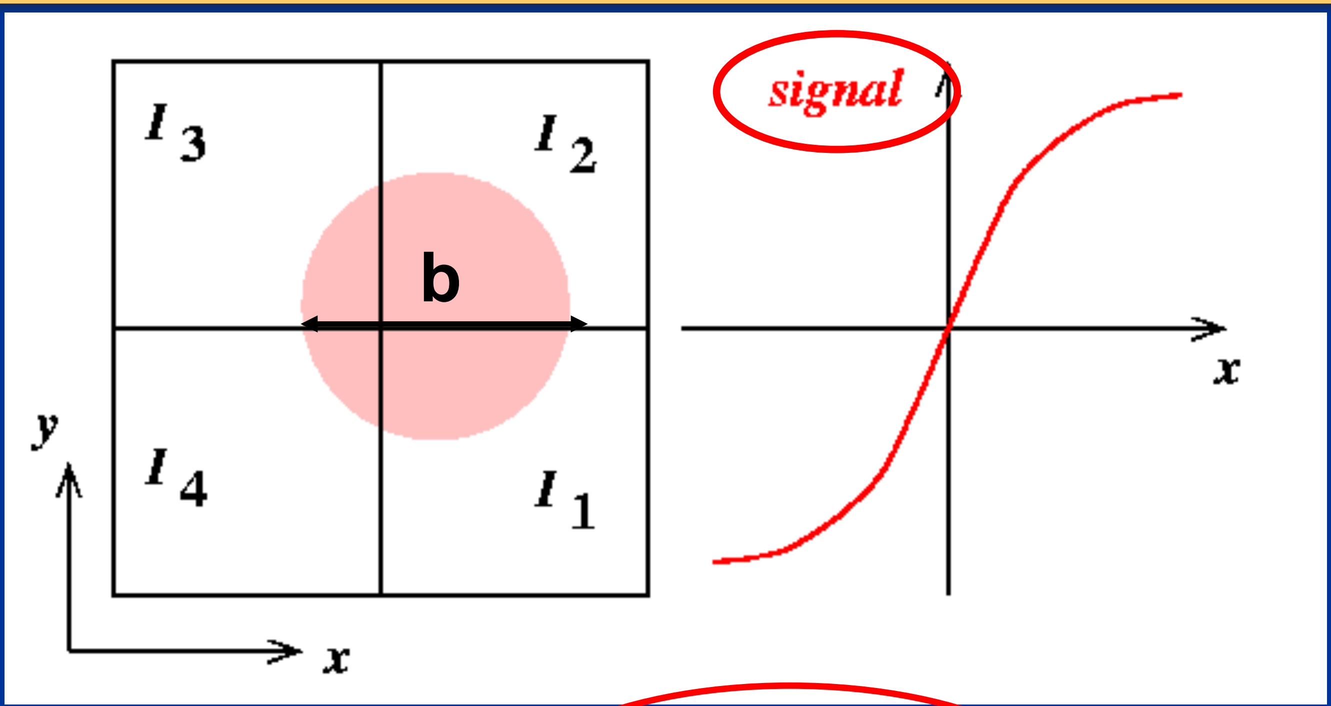
Shack-Hartmann Spots



Wavefront shape

Credit: Thorlabs

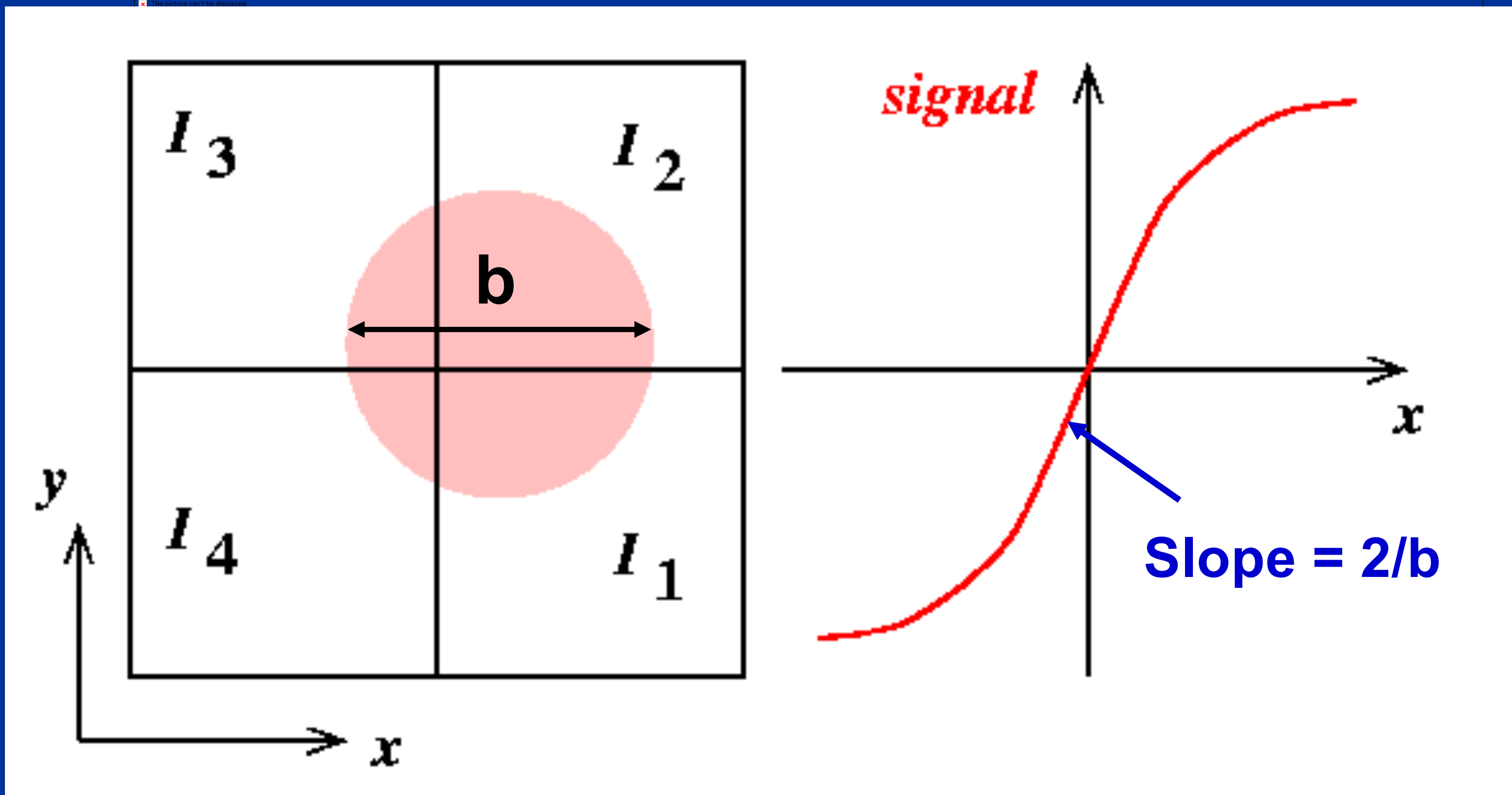
How to measure distance a spot has moved on CCD? "Quad cell formula"



$$\delta_x \cong \frac{b}{2}\left[\frac{(I_2+I_1)-(I_3+I_4)}{(I_1+I_2+I_3+I_4)}\right]$$

$$\delta_y \cong \frac{b}{2}\left[\frac{(I_3+I_2)-(I_4+I_1)}{(I_1+I_2+I_3+I_4)}\right]$$

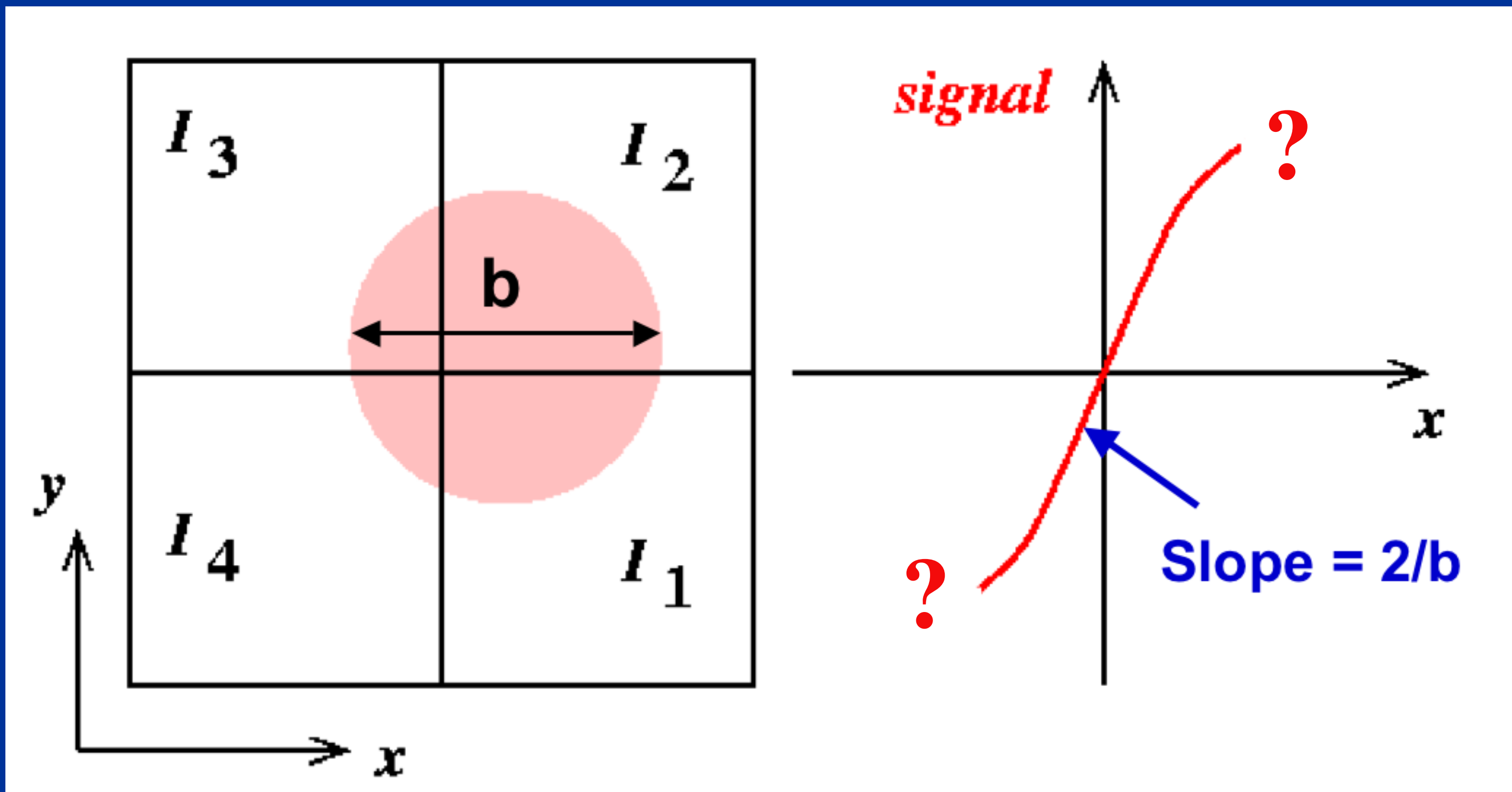
Disadvantage: "gain" depends on spot size b which can vary during the night



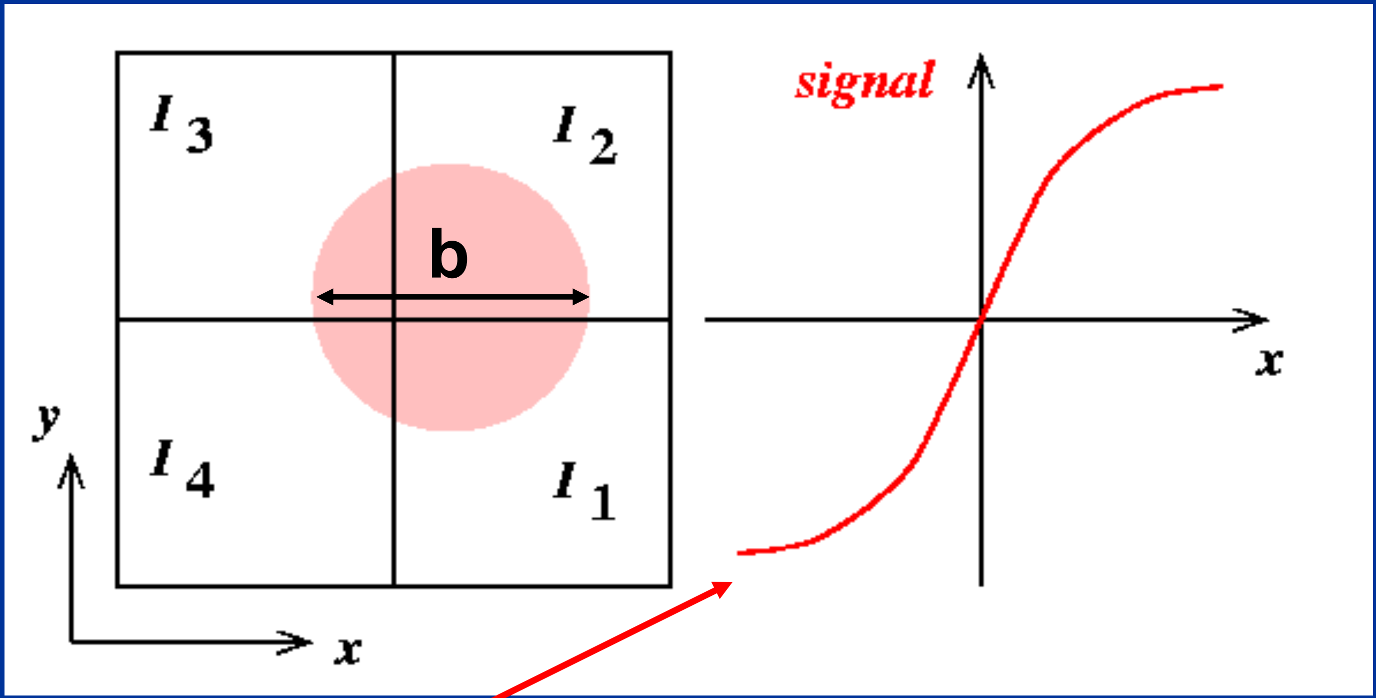
$$\delta_{x,y} = \frac{b}{2}\frac{(difference\ of\ I's)}{(sum\ of\ I's)}$$

Question

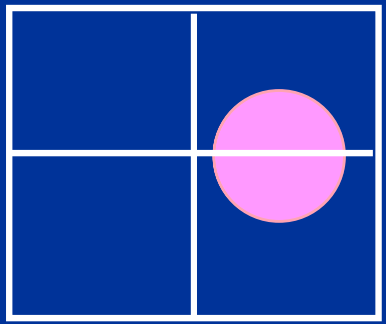
- What might happen if the displacement of the spot is > radius of spot? Why?



Signal becomes nonlinear and saturates for large angular deviations



"Rollover" corresponds to spot being entirely outside of 2 quadrants



Measurement error from Shack-Hartmann sensing

- Measurement error depends on size of spot as seen in a subaperture, θ_b , wavelength λ , subaperture size d, and signal-to-noise ratio SNR:

$$\sigma_{S-H} = \frac{\pi^2}{2\sqrt{2}} \; \frac{1}{SNR} \left[\left(\frac{3\mathrm{d}}{2\mathrm{r}_0} \right)^2 + \left(\frac{\vartheta_b d}{\lambda} \right)^2 \right]^{1/2} \text{ rad} \qquad \text{for } \; r_0 \le d$$

$$\sigma_{S-H} \cong \frac{6.3}{SNR} \text{ rad of phase} \qquad \text{for } \; r_0 = d \text{ and } \vartheta_b = \frac{\lambda}{d}$$

(Hardy equation 5.16)

Order of magnitude, for r_0 ~ d

- If we want the wavefront error to be < $\lambda/20$, we need

$$\Delta z \equiv \frac{\sigma}{k} < \frac{\lambda}{20} \quad \text{or} \quad \sigma \cong \frac{6.3}{SNR} < \frac{2\pi}{20} \text{ so that SNR} > 20$$

General expression for signal to noise ratio of a pixelated detector

- S = flux of detected photoelectrons / subap

  n_{pix} = number of detector pixels per subaperture

  R = read noise in electrons per pixel

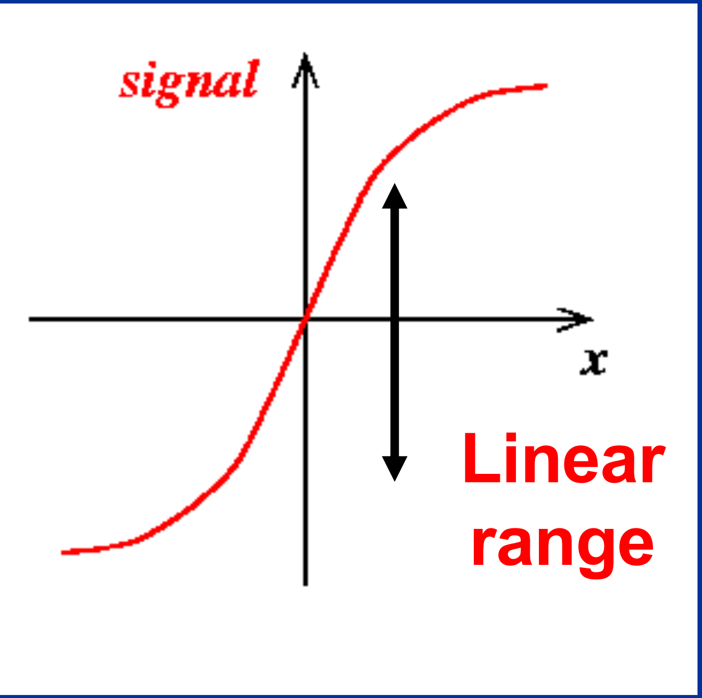
- The signal to noise ratio in a subaperture for fast CCD cameras is dominated by read noise, and

$$SNR \approx \frac{St_{\text{int}}}{(n_{pix}R^2 / t_{\text{int}})^{1/2}} = \frac{S\sqrt{t_{\text{int}}}}{\sqrt{n_{pix}}R}$$

See McLean, "Electronic Imaging in Astronomy", Wiley

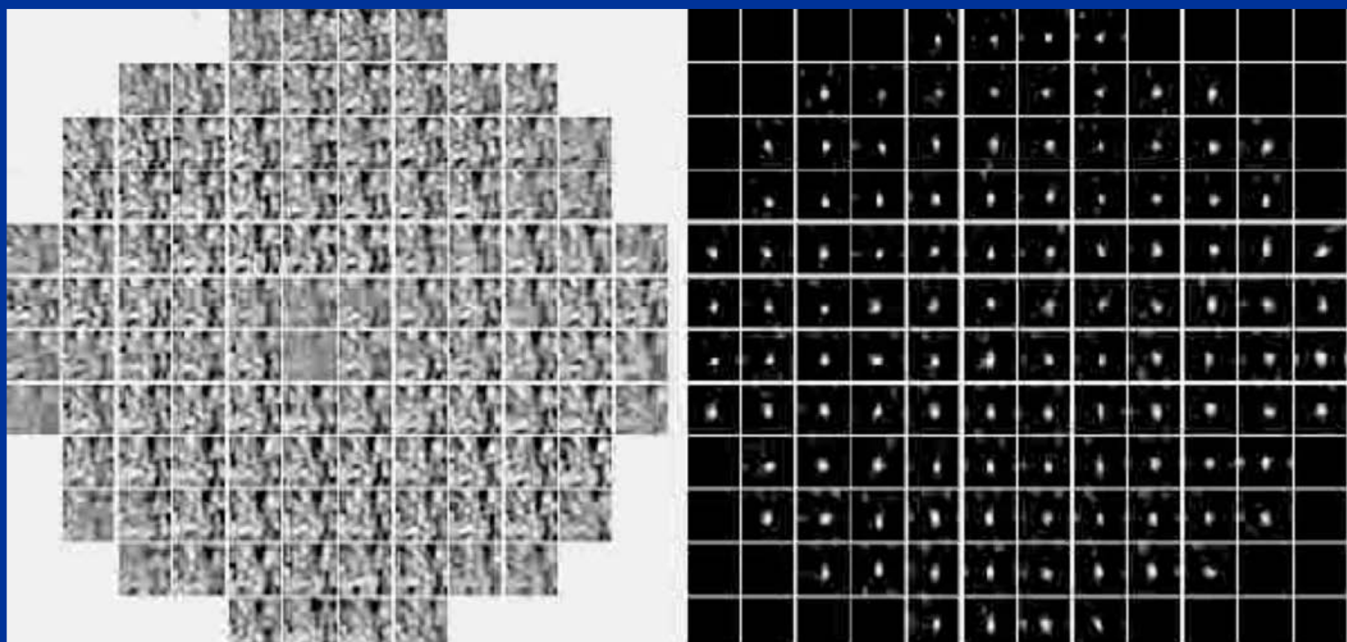
Trade-off between dynamic range and sensitivity of Shack-Hartmann WFS

- If spot is diffraction limited in a subaperture *d*, linear range of quad cell (2x2 pixels) is limited to $\pm \lambda_{ref}/2d$.
- Can increase dynamic range by enlarging the spot (e.g. by defocusing it).
- But uncertainty in calculating centroid $\propto$ width x $N_{ph}^{1/2}$ so centroid calculation will be less accurate.
- Alternative: use more than 2x2 pixels per subaperture. Decreases *SNR* if read noise per pixel is large (spreading given amount of light over more pixels, hence more read noise).



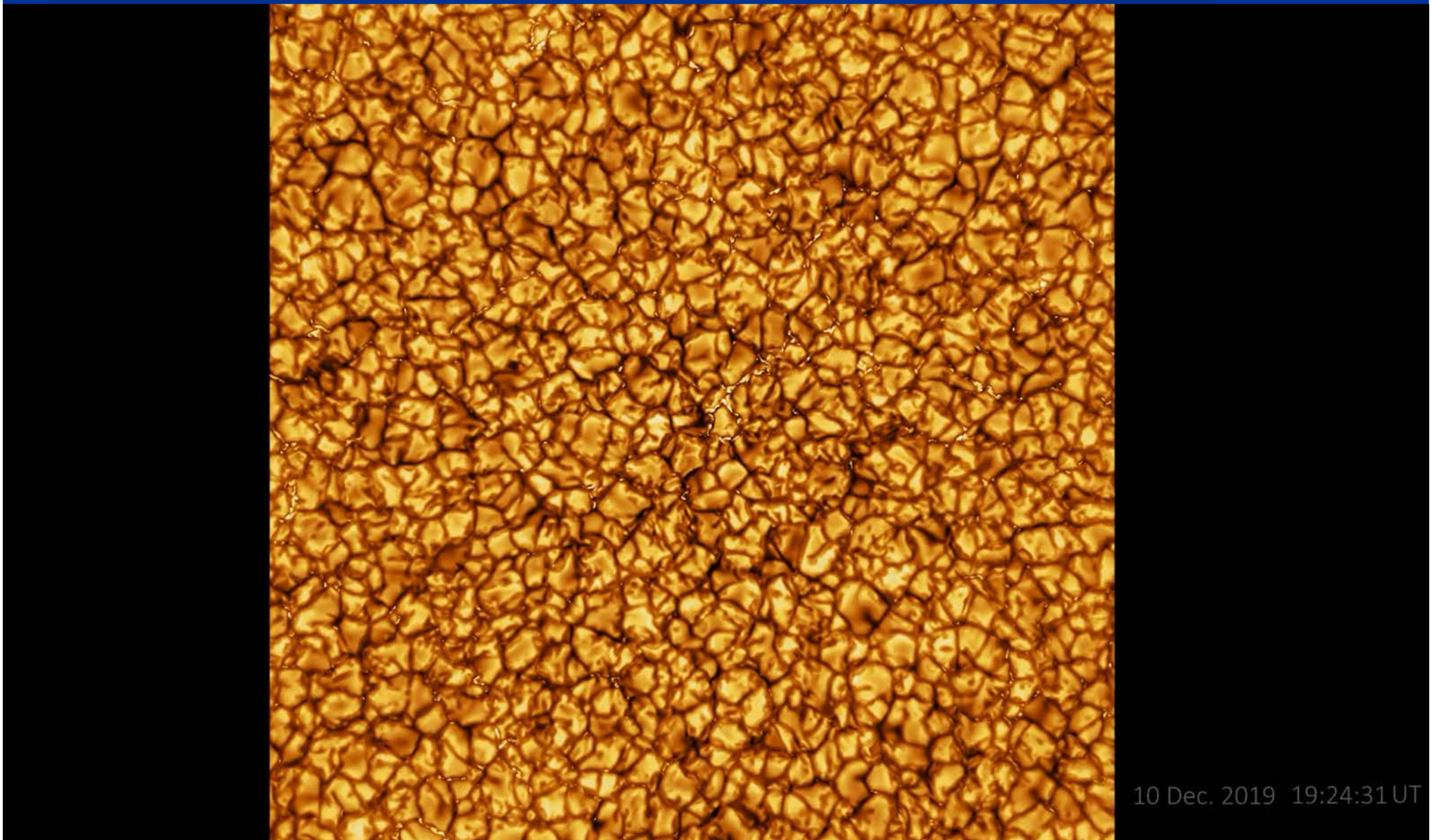
Correlating Shack-Hartmann wavefront sensor uses images in each subaperture

- Solar adaptive optics: Rimmele and Marino, Solar Physics Living Reviews

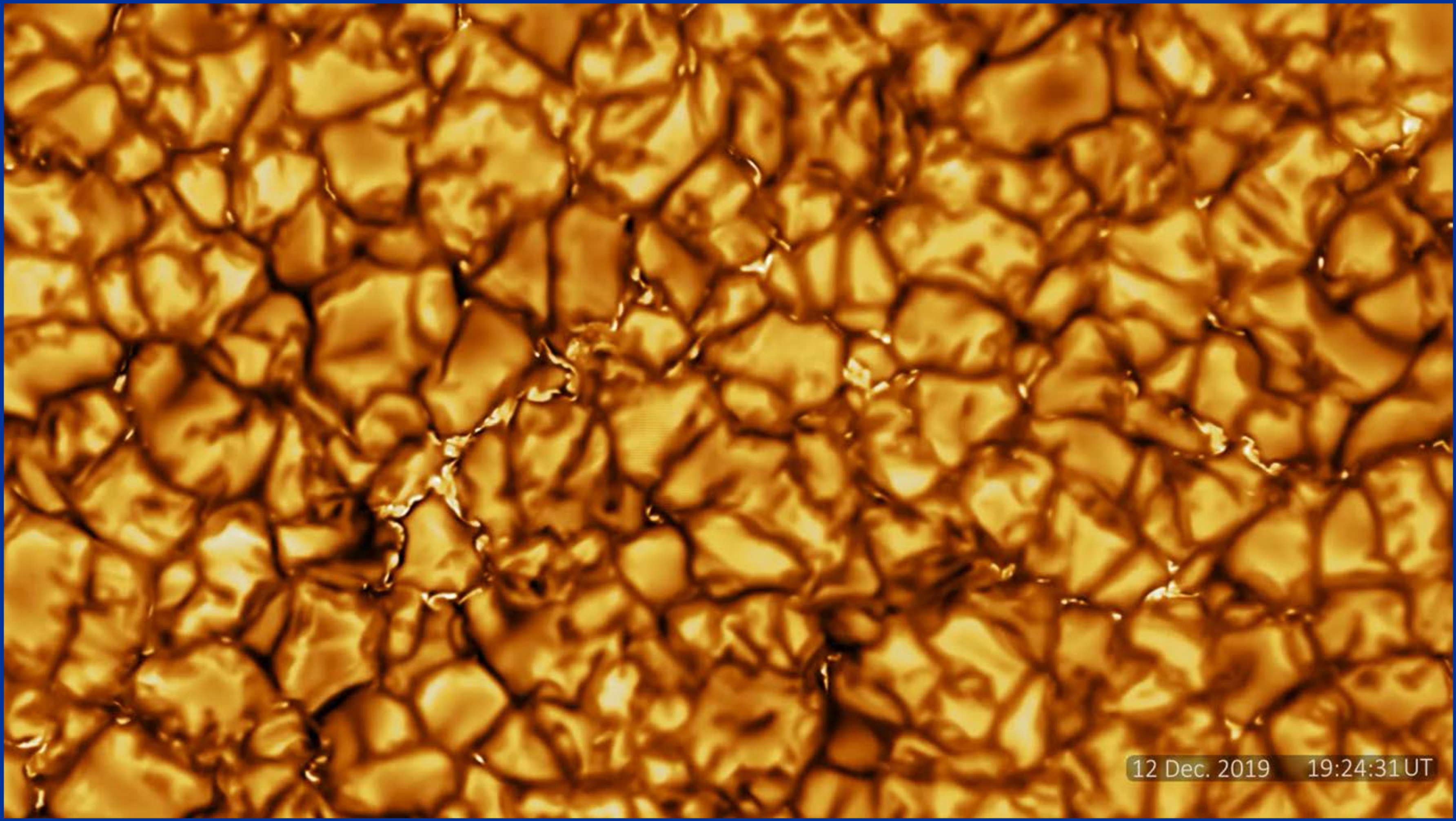


- Cross-correlation is used to track low contrast granulation
- Left: Subaperture images, Right: cross-correlation functions

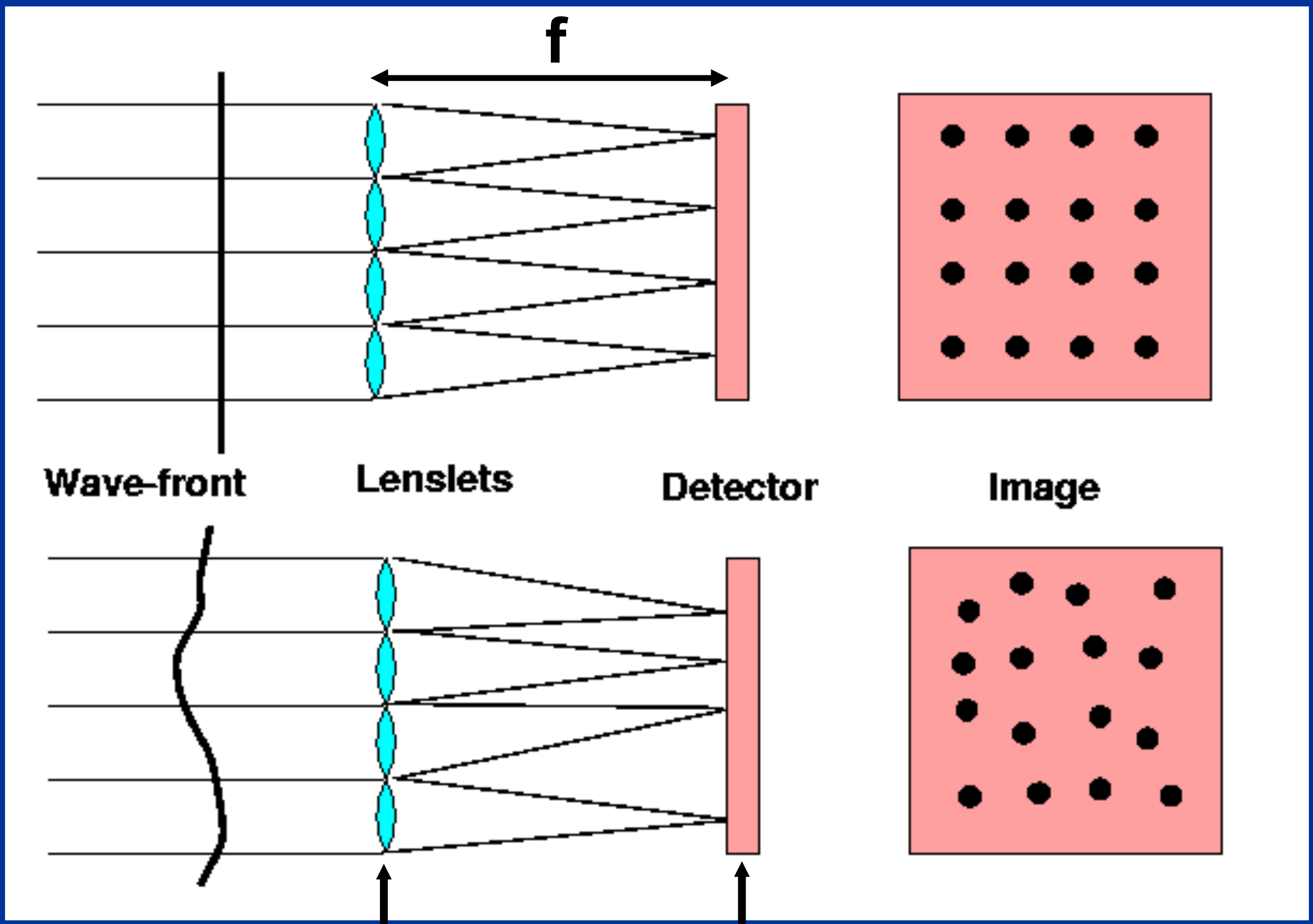
Brand new result: video of Sun from DKIST telescope, with AO



Zoomed-in Version



Review of Shack-Hartmann geometry



Pupil plane

Image plane

Pyramid sensor reverses order of operations in a Shack-Hartmann sensor

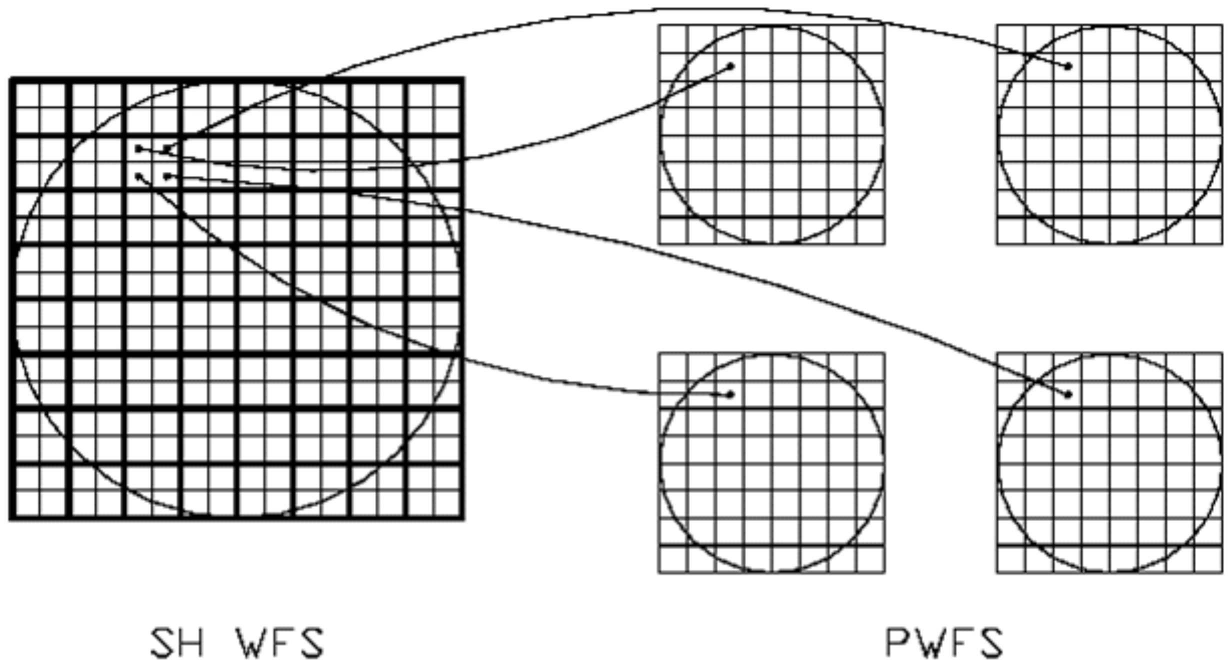
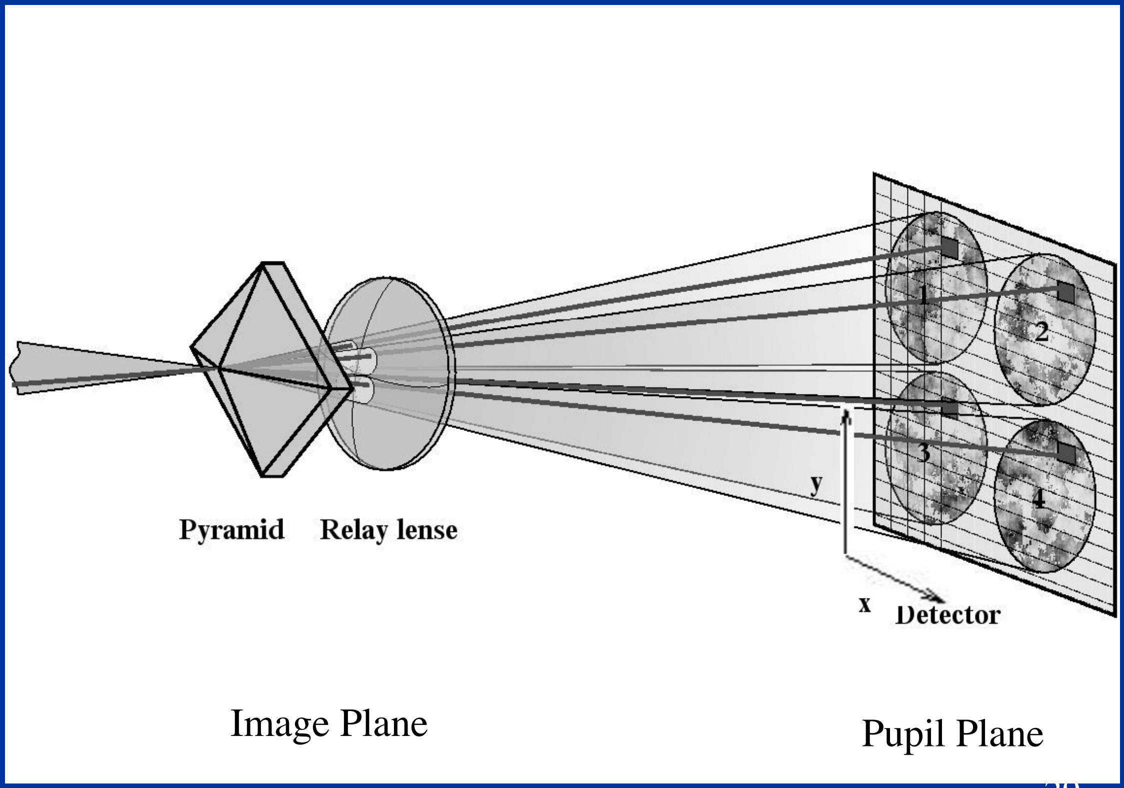


Figure 3- 4: Organization of SH wavefront data (left) versus pyramid wavefront data (right). The circle indicates the beam footprint on the WFS. The heavily-weighted squares on the left indicate the various subapertures (8x8 grid of subapertures). Each subaperture has 4 pixels (a quad cell). In a pyramid wavefront sensing scheme, each pixel represents a subaperture; the 4 images of the pupil correspond to the quadrants of the quad cell.

A Pyramid WFS

Stellar image is placed on the tip of a four sided pyramid --- Creates four beams.

Intermediate optics form pupil images from the four beams.



Credit: Sebastian Egner

Pyramid for the William Herschel Telescope's AO system

Typical intensity patterns for a Pyramid Sensor

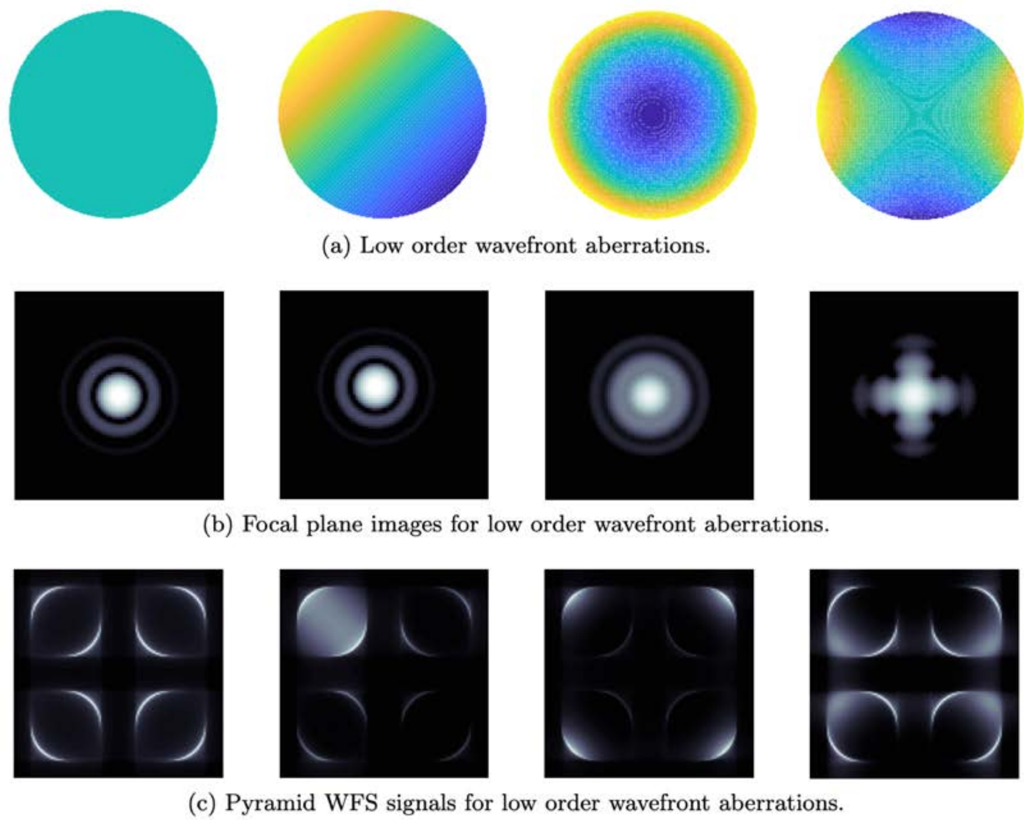
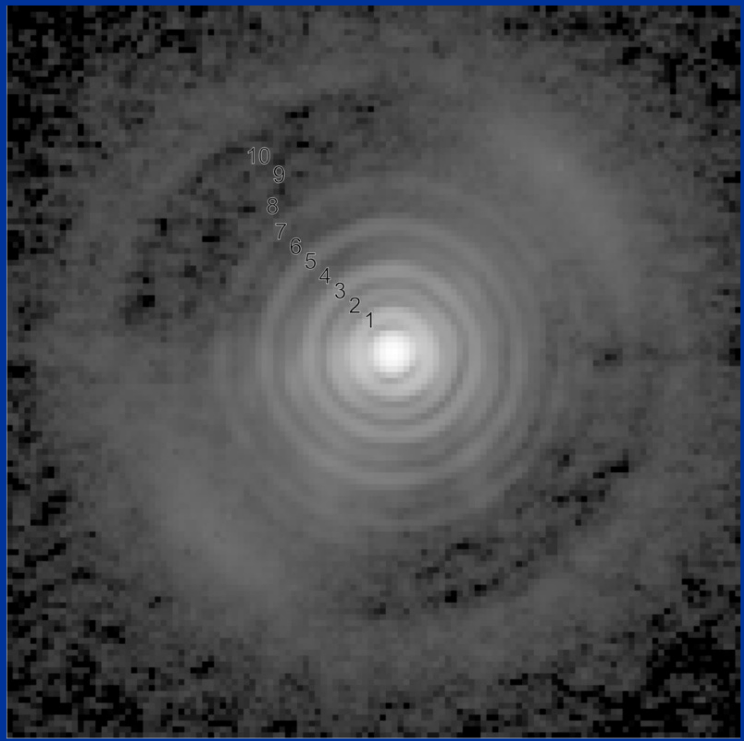


Figure 1: Effect of low order phase aberrations on the wavefront (a), focal plane (b) and Pyramid WFS signal (c). From left to right: flat, tip-tilt, focus, and astigmatism.

Credit: Charlotte Bond

Aspects of Pyramid Sensors

- More sensitive to low order modes.
  - Good match to atmosphere.
  - A change in sampling can easily be carried out via binning of CCD.

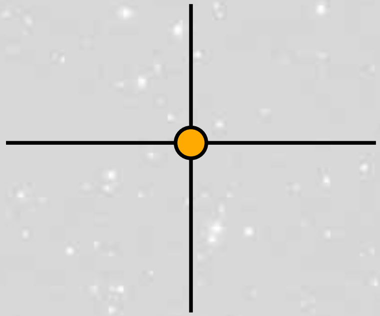


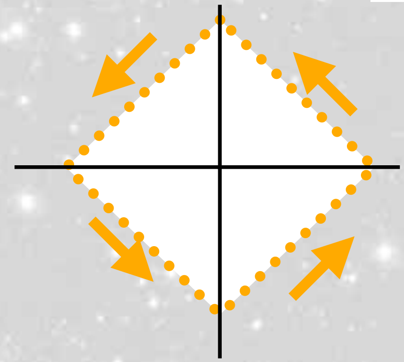
Pyramid Based Systems

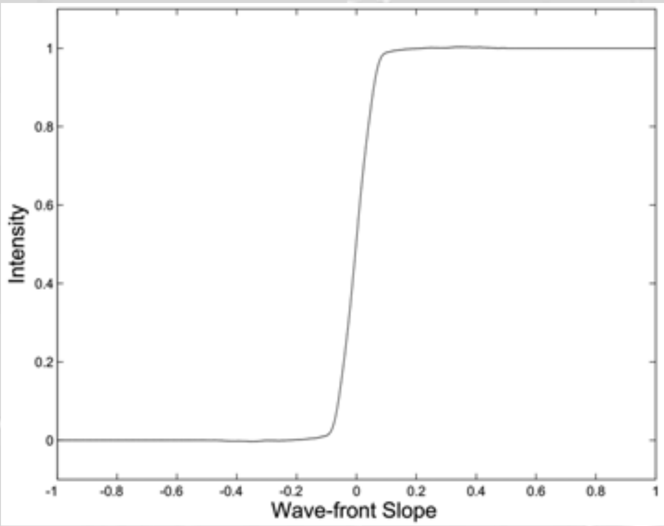
- LBT
- Magellan
- Subaru SCexAO
- Keck IR Pyramid Sensor

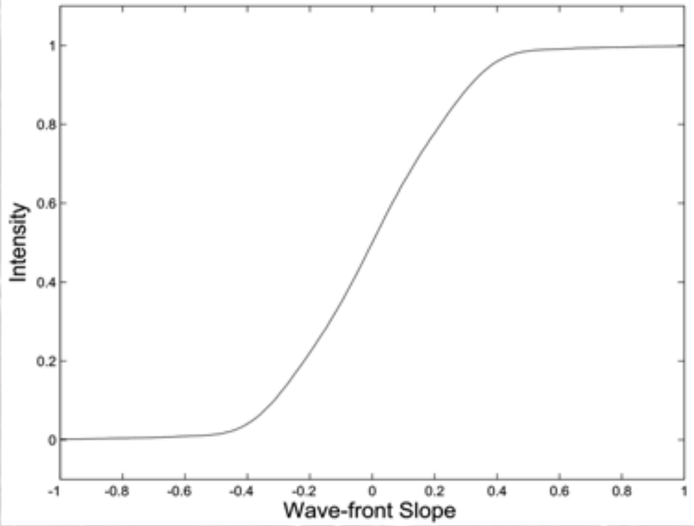
Credit: Phil Hinz

Modulation of pyramid sensor









Without modulation:
Linear over spot width

With modulation:
Linear over modulation width

Credit: Marcos van Dam

Potential advantages of pyramid wavefront sensors

- Wavefront measurement error can be much lower
  - Shack-Hartmann: size of spot limited to λ / d, where d is size of a sub-aperture and usually d ~ r_0
  - Pyramid: size of spot can be as small as λ / D, where D is size of whole telescope. So spot can be D/r_0 = 20 - 100 times smaller than for Shack-Hartmann
  - Measurement error (e.g. centroiding) is proportional to spot size/SNR. Smaller spot = lower error.
- Avoids bad effects of charge diffusion in CCDs
  - Fuzzes out edges of pixels.
  - Pyramid doesn't mind as much as S-H.

Potential pyramid sensor advantages, continued

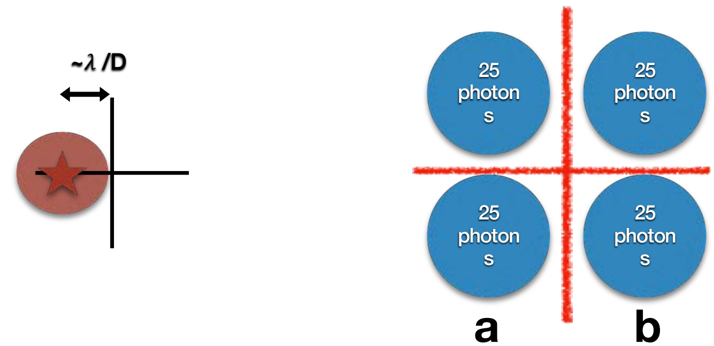
- Linear response over a larger dynamic range
- Naturally filters out high spatial frequency information that you can't correct anyway

A Sensitivity Comparison

- Assume we have I_{total} = 100 photons incident on a wavefront sensor.
- Assume an ideal detector (only photon noise affects precision).
- Compare a 20x20 S-H sensor to a pyramid sensor for measuring tilt.

S-H

- ~300 subapertures
- 0.3 photons per sub aperture
  - SNR=0.3/√0.3 = 0.54
- PSF = λ/d = 20 λ /D
- 300 independent measurements (N=300)
- σ_{tilt} = FWHM / SNR / √N = 2.1 λ /D



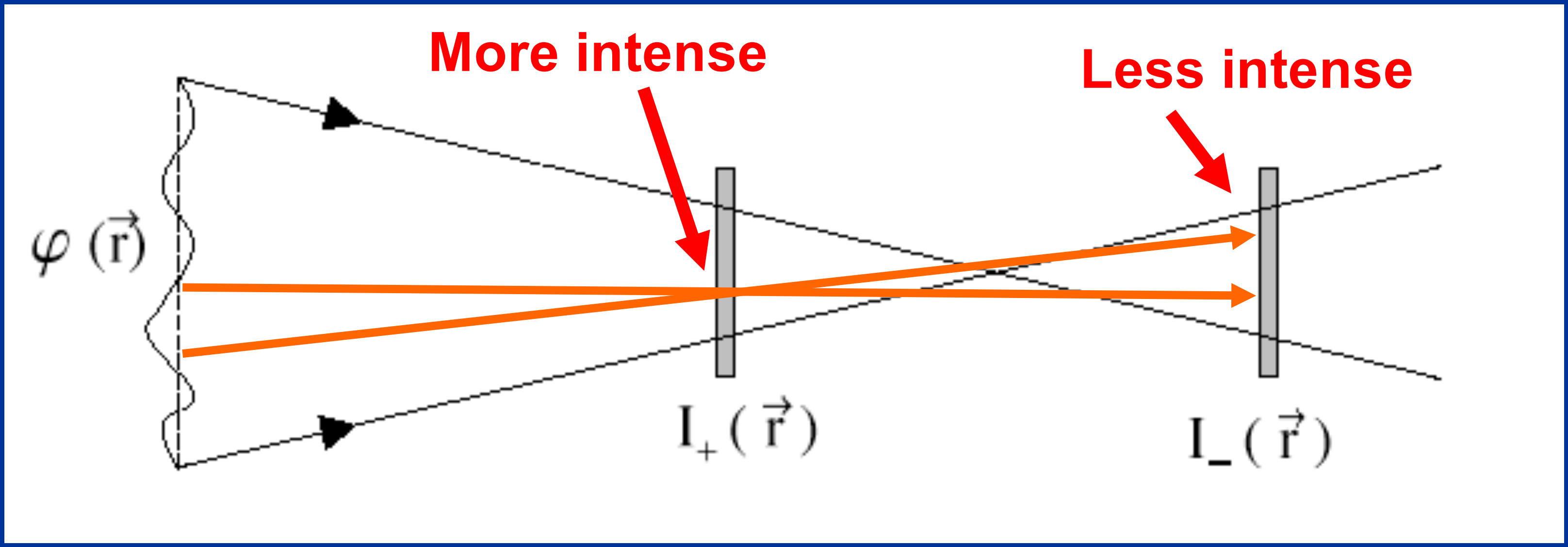
Pyramid

- Pyramid has all flux to one side (two pupils) when tilt offset is ~1 λ/D
  - $(I_a - I_b)/I_{total} = 1$ -> tilt = $(I_a - I_b)/I_{total}$ * λ/D
- There are, on average 50 photons on each side to measure flux balance
  - σ_{tilt} = [√ (50+ 50) /100]* λ/D = 0.1 * λ/D

Credit: Phil Hinz

Curvature wavefront sensing

- F. Roddier, Applied Optics, 27, 1223- 1225, 1998



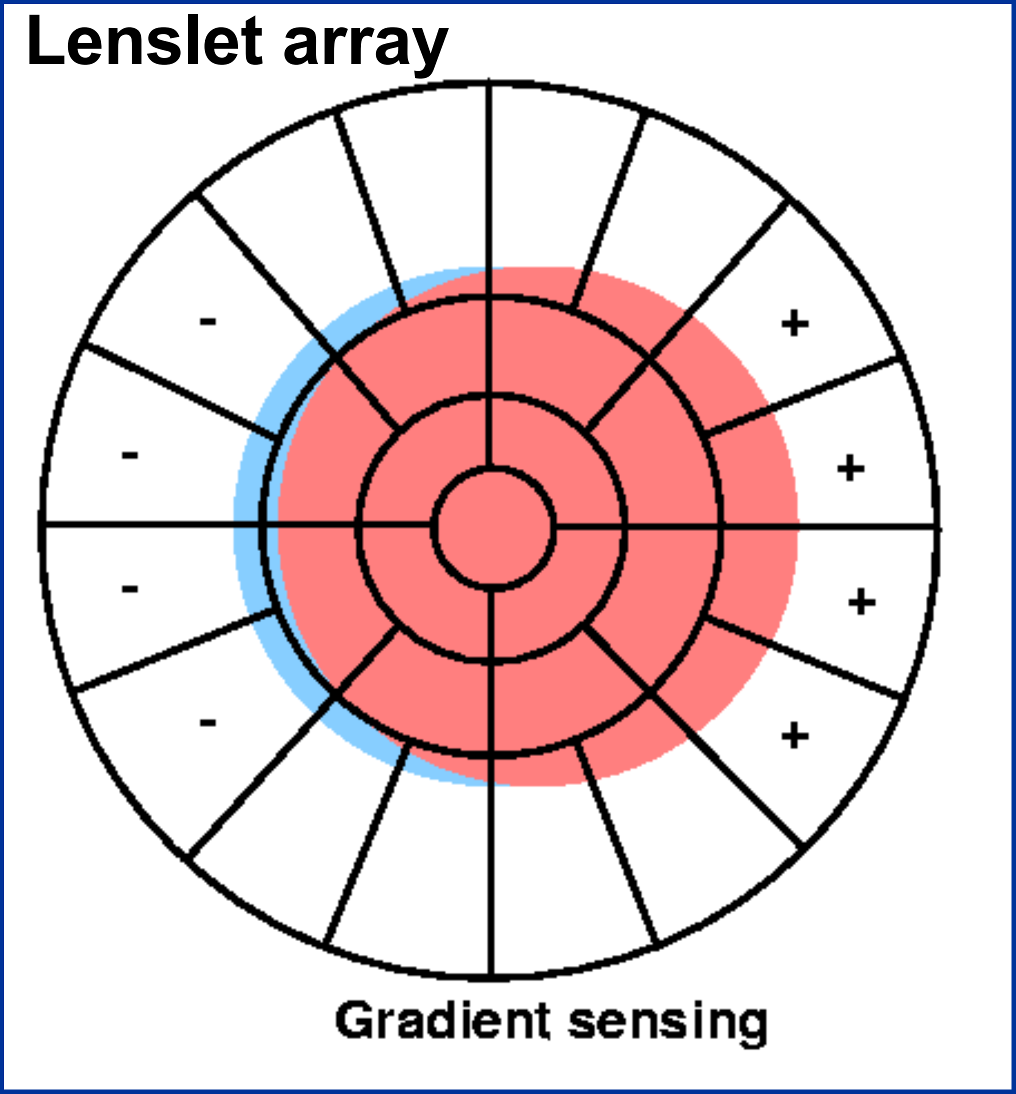
$$\frac{I_+ - I_-}{I_+ + I_-} \propto \nabla^2\phi - \frac{\partial\phi}{\partial\vec{r}}\vec{\delta}_R$$

Normal derivative at boundary

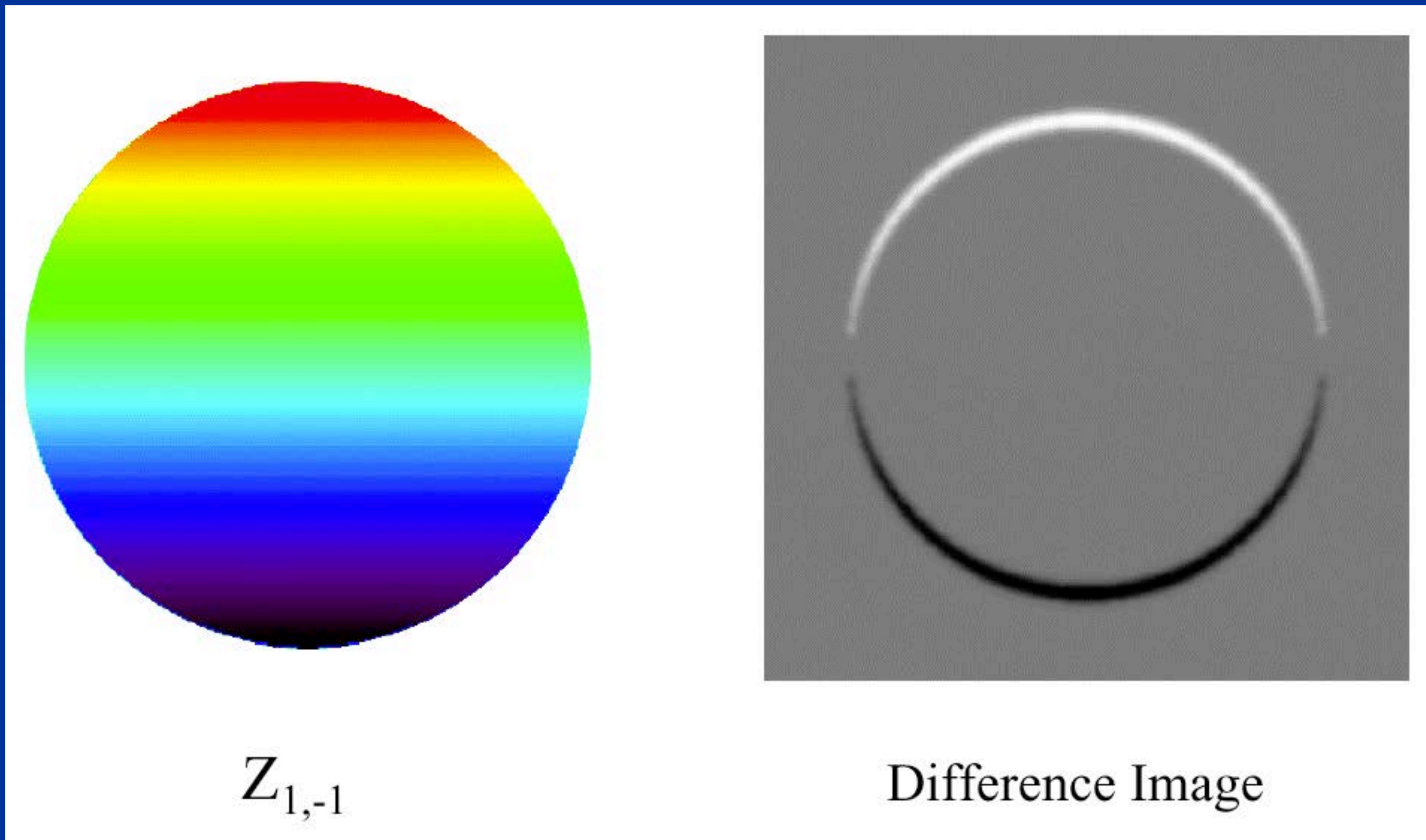
Laplacian (curvature)

Wavefront sensor lenslet shapes are different for edge, middle of pupil

- Example: This is what wavefront tilt (which produces image motion) looks like on a curvature wavefront sensor
  - Constant I on inside
  - Excess I on right edge
  - Deficit on left edge



Simulation of curvature sensor response

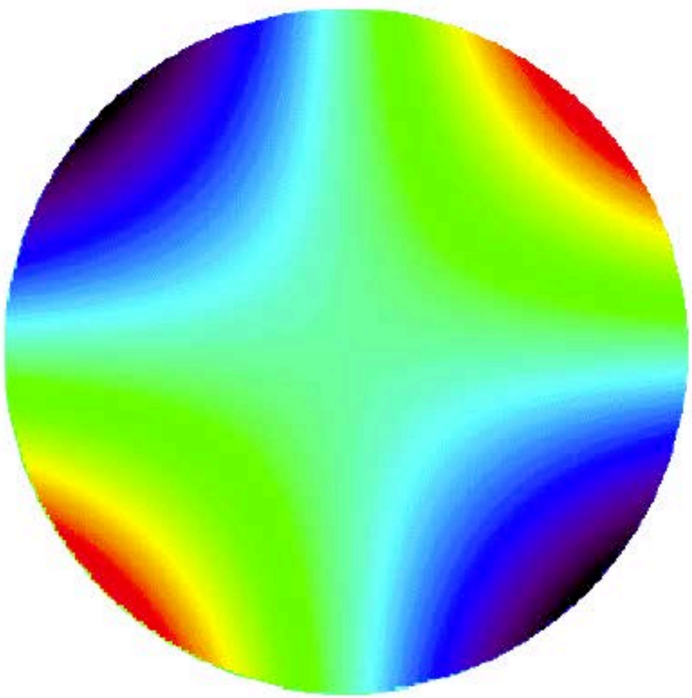


Wavefront: pure tilt

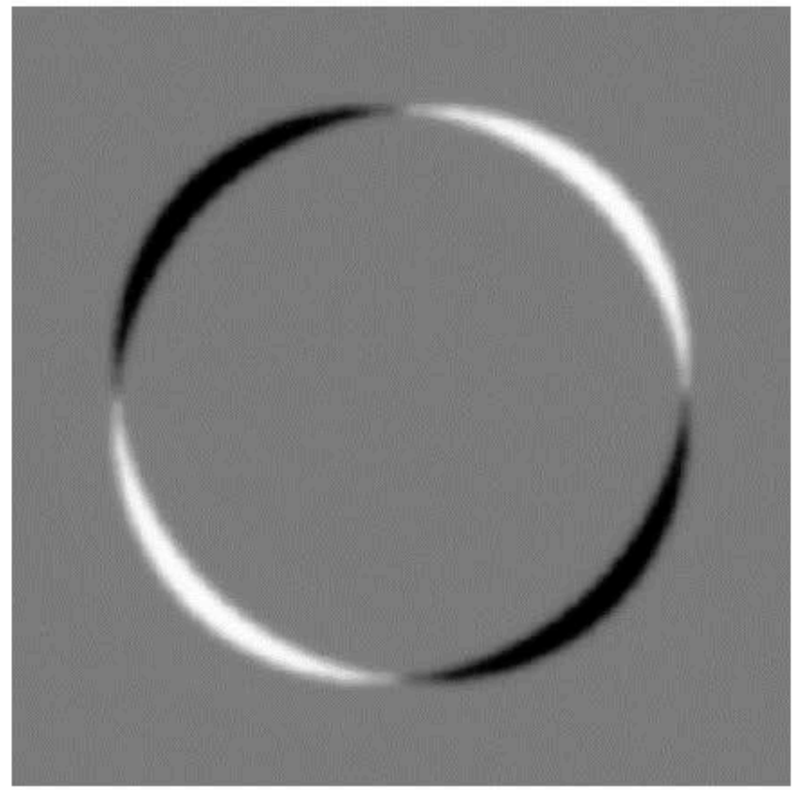
Curvature sensor signal

Credit: G. Chanan

Curvature sensor signal for astigmatism



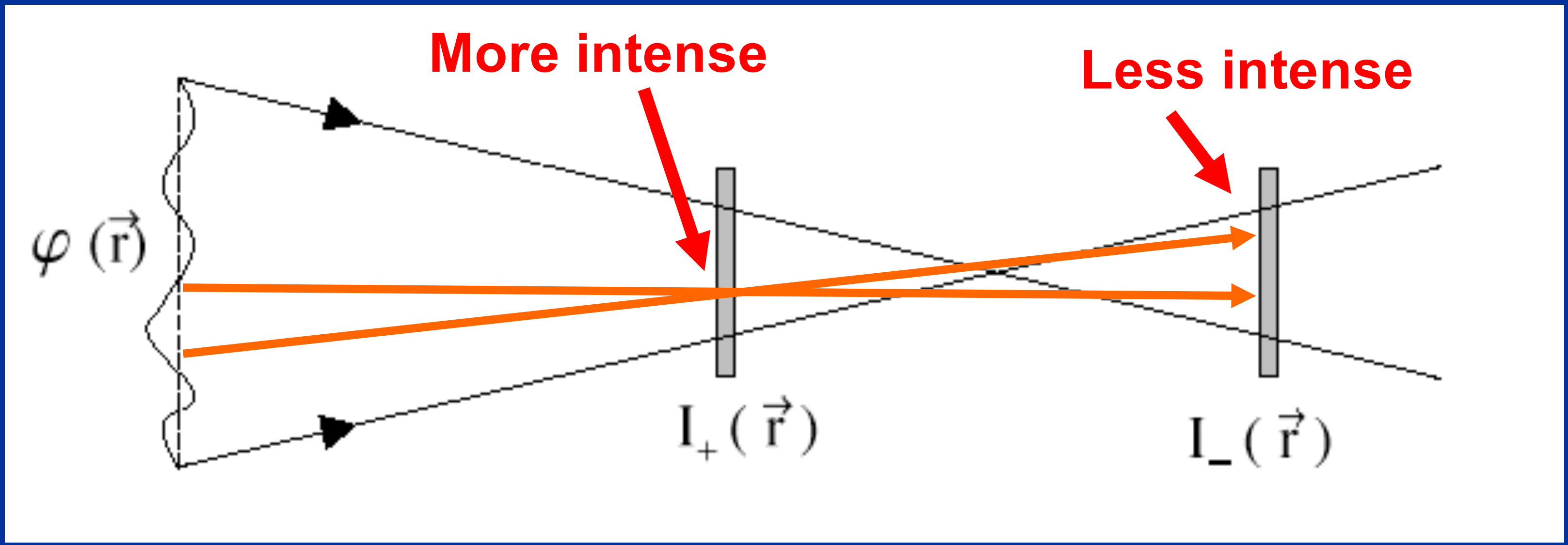
$Z_{2,-2}$



Difference Image

Credit: G. Chanan

Practical implementation of curvature sensing



- Use oscillating membrane mirror (2 kHz!) to vibrate rapidly between I_+ and I_- extrafocal positions
- Measure intensity in each subaperture with an "avalanche photodiode" (only need one per subaperture!)
  - Detects individual photons, no read noise, QE ~ 60%
  - Can read out very fast with no noise penalty

Measurement error from curvature sensing

- Error of a single set of measurements is determined by photon statistics, since detector has NO read noise!

$$\sigma_{cs}^2 = \pi^2 \frac{1}{N_{ph}} \left(\frac{\theta_b d}{\lambda} \right)^2$$

where d = subaperture diameter and N_{ph} is no. of photoelectrons per subaperture per sample period

- Error propagation when the wavefront is reconstructed numerically using a computer scales poorly with no. of subapertures N:

$(\text{Error})_{\text{curvature}} \propto N$, whereas $(\text{Error})_{\text{Shack-Hartmann}} \propto \log N$

Advantages and disadvantages of curvature sensing

- Advantages:
  - Lower noise ⇒ can use fainter guide stars than S-H
  - Fast readout ⇒ can run AO system faster
  - Can adjust amplitude of membrane mirror excursion as "seeing" conditions change. Affects sensitivity.
  - Well matched to bimorph deformable mirror (both solve Laplace's equation), so less computation.
  - Curvature systems appear to be less expensive.
- Disadvantages:
  - Avalanche photodiodes can fail with too much light
  - Hard to use a large number of avalanche photodiodes.
  - BUT – recently available in arrays
  - Doesn't scale well to large numbers of subapertures

Summary of main points

- Wavefront sensors in common use for astronomy measure intensity variations, deduce phase. Complementary.
  - Shack-Hartmann
  - Curvature sensors
- Curvature systems: cheaper, fewer degrees of freedom, scale more poorly to high no. of degrees of freedom, but can use fainter guide stars
- Shack-Hartmann systems excel at very large no. of degrees of freedom
- Most recent addition: pyramid sensors
  - Very successful for faint natural guide stars, low modes